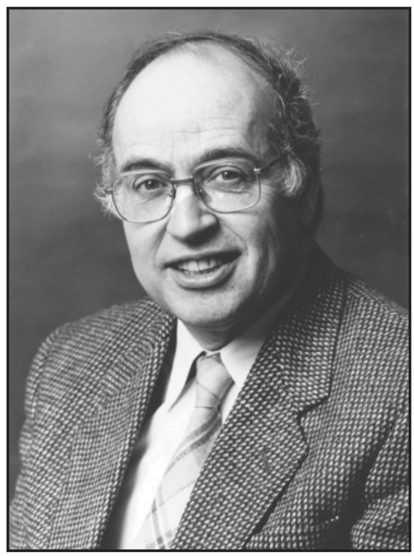
MICHAEL FRANCIS ATIYAH



22 APRIL 1929 · 11 JANUARY 2019

Michael Francis Atiyah was one of the most influential mathematicians of the last century. His work helped reshape multiple areas of mathematics—notably analysis, geometry, and topology. He forged new and unexpected connections between different fields. Beginning in the 1970s, Atiyah also played a major role in redefining the relationship between mathematics and physics.

Beyond his scientific work, Atiyah was prominent in scientific leadership and in public affairs. Among other commitments, he was president of the Royal Society of London from 1990 to 1995, president of the Pugwash Conferences on Science and World Affairs from 1997 to 2002, and president of the Royal Society of Edinburgh from 2005 to 2008.

Atiyah's father, Edward Selim Atiyah, was an Anglo-Lebanese author and political activist. His mother, Jean Levens, was Scottish. Atiyah's primary school education was at the Diocesan school in Khartoum, Sudan, and he attended secondary school in Cairo and Alexandria during the World War II years. His undergraduate and graduate education were at the University of Cambridge in the United Kingdom. His doctoral thesis was written at Cambridge under the supervision of William V. D. Hodge, himself a pioneer of some of the connections between topology and analysis.

Atiyah's early interests were in applications of topology to algebraic geometry—that is, he was interested in understanding the complicated spaces that can be defined by the solutions of simple equations. In the late 1950s, with Friedrich Hirzebruch, Atiyah created a new tool known as topological K-theory. To do this, they took an idea from algebraic geometry and adapted it for the more flexible purposes of topology. Atiyah went on to show that topological K-theory is a powerful tool that can be used to give simple and illuminating solutions to what previously were rather difficult problems.

In its original context of algebraic geometry, K-theory had been invented by Alexander Grothendieck, who had used it as the foundation for a vast generalization of a celebrated 19th-century theorem of Bernhard Riemann and Gustav Roch. Atiyah came to suspect that there should exist an analog in topology of the Grothendieck-Riemann-Roch theorem of algebraic geometry. For several years he searched for such an analog. This search was eventually crowned with success in the Atiyah-Singer index theorem, originally proved in 1963 by Atiyah with Isadore Singer. This was Atiyah's single most important work.

Roughly, the index theorem predicts the number of independent solutions of a differential equation in terms of the shape, or more precisely the topology, of the space on which the equation is defined. It links multiple areas of mathematics and even physics in a most remarkable way. The statement and proof involve calculus and differential equations. An important input comes from physics, because a key discovery of Atiyah and Singer in formulating and proving their theorem was that the Dirac equation of relativistic electron theory plays a central role. And the implications of the theorem are widely felt in different areas of mathematics, especially geometry and topology.

In the following decade, Atiyah developed many extensions and applications of the index theorem, in work with Graeme Segal, Raoul Bott, and Vijay Patodi, among others. His papers from this period opened up many new fields of research, with repercussions in multiple directions. In the mid-1970s, Atiyah developed a serious interest in physics, and thereafter he became an influential figure in theoretical physics. What set the stage for Atiyah's interest in physics was the emergence by the mid-1970s of the Standard Model of particle physics. Physicists in the 1950s and 1960s had been preoccupied with problems of quantum theory that seemed far removed from the concerns of modern mathematics. I, for example, as a physics graduate student of the 1970s, did not learn about the Atiyah-Singer index theorem or any other topic in modern mathematics. Conversely, mathematicians of the period were generally not in close touch with developments in physics. For instance, Atiyah and Singer, when they needed the Dirac equation, had to reinvent it, as Atiyah recounted in later reminiscences.

However, the emergence of the Standard Model changed the picture. In trying to understand the Standard Model, physicists asked new questions that called for greater mathematical sophistication, though this was realized only gradually. Atiyah, Singer, and other mathematicians who became interested in what physicists were doing played an important role in this process.

By 1976, physicists had discovered novel and mathematically interesting "instanton" solutions of the equations of the Standard Model. Moreover, an important puzzle about the Standard Model had been solved by using unusual properties of the Dirac equation interacting with these instantons. Soon it was understood (originally by Albert Schwarz) that the key facts here were consequences of the Atiyah-Singer index theorem. It was this that made the index theorem and the names Atiyah and Singer widely familiar to physicists.

I myself first met Atiyah in the spring of 1977 when he visited MIT, invited by Roman Jackiw, a physicist whose work had helped catalyze some of the developments mentioned above. At the time, Atiyah was explaining his work with Richard Ward, showing that the instanton equation could be solved using the "twistor transform" of Roger Penrose. His lectures had a big impact on the mathematics and physics communities in Cambridge, Massachusetts. Physicists at the time were very interested in solving the instanton equation, and Atiyah had made a lot of progress with this in his work with Richard Ward. But the ingredients in Atiyah's approach—complex manifolds, sheaf cohomology, fiber bundles—were quite unfamiliar to me and to most of the other physicists. He was introducing us to an entirely new world.

By January 1978, when Atiyah invited me to visit Oxford for a few weeks, he was lecturing at the Mathematical Institute about a more precise understanding of instantons, which he had achieved with Vladimir Drinfeld, Yuri Manin, and his student Nigel Hitchin. Toward the end of my visit, he showed me two physics papers that I had not seen before. By this time it was known that gauge theories, similar to the Standard Model, can describe magnetic monopoles as well as ordinary elementary particles. Peter Goddard, Jean Nuyts, and David Olive (GNO) had suggested that magnetic charges are classified by a "dual gauge group," and Olive with Claus Montonen had gone on to suggest a specific symmetry between electricity and magnetism. Atiyah observed that the GNO dual gauge group was the same as the dual group introduced by Robert Langlands in number theory. The Langlands program is a central but mysterious concept in number theory, which still today is far from fully understood. Atiyah's conception was that somehow the Langlands program would be tied up with symmetry between electricity and magnetism in gauge theory. At the time, it seemed easy to give technical reasons that this could not possibly be right. But over the intervening decades, large pieces of Atiyah's vision have been vindicated, though the full scope of this relationship between number theory and physics is probably still far out of sight.

During the following decade, Atiyah, together with collaborators such as Bott, helped educate me and other physicists about a number of unfamiliar mathematical topics, including Morse theory and equivariant cohomology. They discovered some fascinating results that reduced the gap that had to be bridged to apply these topics to physics. In 1987, Atiyah twice visited the Institute for Advanced Study (IAS) in Princeton, where I was a professor. Among other things, he was excited about a theory of three-dimensional spaces developed by the young mathematician Andreas Floer. He thought that this theory should be reformulated in the framework of quantum theory, and he set to me the problem of doing so. At first this seemed too optimistic, but eventually it became clear that a simple "twist" of the supersymmetric field theories studied by physicists could give a theory with the right properties.

The other problem that Atiyah posed during this visit was to find a quantum field theory interpretation of the Jones polynomial of a knot.

The Jones polynomial is a remarkable invariant of a knot in three-dimensional space that had been discovered a few years earlier by the mathematician Vaughan F. R. Jones. I had never heard of the Jones polynomial before Atiyah recommended this problem, and that certainly put me in the majority among physicists. However, in the years 1987–1988 there was a flood of new results with bearing on the Jones polynomial, including in work by some of the younger physicists at the IAS. Ultimately it turned out that the Jones polynomial could indeed be understood in the language of physics, with the knot interpreted as the trajectory of a charged particle in a world of three spacetime dimensions.

In the spring of 2001, Atiyah visited for several months at Caltech, where I was on sabbatical. We had a memorable collaboration on a problem involving string theory and geometry.

Michael Atiyah played an enormous role in introducing new ideas and encouraging and teaching physicists to study quantum theory from new points of view. It took many twists and turns for these lessons to be really learned and absorbed in the physics world. Atiyah always believed that the study of quantum theory as a tool in geometry had to be integrated with more "physical" aspects of the study of quantum theory. His vision and clairvoyance have had a truly far-reaching influence.

Elected 1991

EDWARD WITTEN
Charles Simonyi Professor
School of Natural Sciences, Institute for Advanced Study