Did Einstein Get It Right? A Centennial Assessment[a]

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I. Introduction

The year 2015 marked the 100th anniversary of the publication of Einstein's general theory of relativity, and relativists worldwide celebrated this historic occasion. As if this were not enough, on September 14, 2015, scientists at the LIGO gravitational-wave observatories in the United States detected, for the first time, gravitational waves passing the Earth, emitted by a pair of merging black holes more than a billion light years away. This event provided a kind of fairy-tale close to a remarkable century.

Indeed, some popular accounts of the history of general relativity read like a fairy tale, going something like this: in 1905, Einstein discovered special relativity. He then turned his attention to general relativity, and after 10 years of hard work, he got general relativity in November 1915. In 1919, Eddington verified the theory by measuring the bending of starlight. Einstein became famous. And everybody lived happily ever after.

The real history of general relativity is rather more complex. At the time of Eddington's measurements of light bending, there was considerable skepticism about the results, especially among American astronomers. There were major conceptual difficulties with the theory; it was hard to understand what this new theory was and what it really predicted. And finally, there was an abiding sense, notwithstanding its radical conception of gravity as geometry, that the theory mainly predicted some tiny corrections to Newtonian gravity and really wasn't all that important for physics.

As a result, within about 10 years of its development, general relativity entered a period of decline, dubbed the "low-water mark" by Jean Eisenstaedt,[1] so that by the end of the 1950s, general relativity was considered to be in the backwaters of physics and astronomy, an unfit subject for a serious scientist to pursue.

a Read 29 April 2016.

But during the 1960s, there began a tremendous renaissance for the theory, fueled in part by the discoveries by astronomers of quasars, pulsars, the cosmic background radiation, and the first black-hole candidate, systems in which it became clear that general relativity would play a crucial role. It was also fueled by the beginnings of a worldwide effort to put the theory to the test using new precision tools such as atomic clocks and radio telescopes, together with the emerging space program.

Today, general relativity is fully integrated into the mainstream of physics and, in fact, is central to some of the key scientific questions of today, such as: How did the universe begin and what is its future? What governs physics at the highest energies and shortest distances? Do black holes really exist, and how do they affect their surroundings? What does general relativity tell us about the most catastrophic and energetic events in the universe?

This paper reviews the empirical foundations of Einstein's great theory.[2] We will address some of the early challenges and uncertainties in testing the theory and provide some selected highlights of the high-precision tests that have been done, mostly from the period after the "renaissance" of the 1960s. We will also indicate opportunities for future tests of general relativity. The discussion will be centered around four main phenomena associated with the spacetime geometry that is the central concept of Einstein's theory, namely that it bends light, it warps time, it moves mass, and it makes waves.

II. Geometry Bends Light

This phenomenon brings to mind the celebrated 1919 measurement of the bending of starlight. On March 8, 1919, just four months after the armistice ending World War I, two expeditions set sail from England: Arthur Stanley Eddington's for the island of Principe, off the coast of present-day Equatorial Guinea, and Andrew Crommelin's for the city of Sobral, in northern Brazil, to observe the solar eclipse of May 29. The principle of the experiment is deceptively simple. During a total solar eclipse, the Moon hides the Sun completely, revealing the field of stars around it. Using a telescope and photographic plates, the astronomers take pictures of the obscured Sun and the surrounding star field. These pictures are then compared with pictures of the same star field taken when the Sun is absent. The comparison pictures are taken at night, several months earlier or later than the eclipse, when the Sun is nowhere near that part of the sky and the stars are in their true, undeflected positions. In the eclipse pictures, the stars whose light is deflected would appear to be displaced away from the Sun relative to their actual positions. Because the deflection varies inversely as the angular distance of the star's image from the Sun, stars far in angle from the Sun establish the fixed reference points for comparing the sets of plates. The maximum deflection for a ray grazing the surface of the Sun is 1.75 arcseconds (an arcsecond is 1/3600 of a degree, or the angle subtended by a human finger at a distance of about 4 kilometers).

When Eddington announced at the November 6, 1919 meeting of the Royal Society of London that his results confirmed Einstein's theory,[3] it caused an international sensation. The headline in the *London Times* of November 7, 1919 read "Revolution in Science/New Theory of the Universe/Newtonian Ideas Overthrown." The following day, *The New York Times* proclaimed "Lights All Askew in the Heavens/Men of Science More or Less Agog over Results. . . . "

Before this, Einstein had been an obscure Swiss/German scientist, well known and respected within the small European community of physicists but largely unknown to the outside world. With the announcement of the measurement of the deflection, aided by some adroit advance publicity engineered by Eddington, all this changed, and Einstein and his theory became immediate sensations.

But there were doubts about these results, particularly among American astronomers.[4] William Campbell and Heber Curtis of the Lick Observatory analyzed plates from a 1900 eclipse in Georgia and a 1918 eclipse in Washington State in the United States and found no deflection; ironically, they reported this negative result at the Royal Society of London meeting in July 1919 in the midst of Eddington's data analysis. (At the meeting, rumors were already going around that Eddington would report a positive result.) Later eclipse measurements—three in 1922, one in 1929, two in 1936, one each in 1947 and 1952, and one in 1973—did offer some support for general relativity. However, there was little improvement in accuracy, with different measurements giving values anywhere between three-quarters and one and one-third times the general relativistic prediction, though there was little doubt about Einstein beating Newton.[5]

The real revolution in measuring the deflection of light came with the development of radio astronomy, which now has the capability to measure angular separations and changes in angles to accuracies better than 100 microarcseconds. Since 1969, measurements of the deflection of radio waves from quasars and radio galaxies have been carried out with steadily increasing precision, today reaching a part in 10,000, in agreement with Einstein's theory. In fact, using worldwide arrays of linked radio telescopes in a technique known as Very Long Baseline Interferometry (VLBI), radio astronomers now can see that the "fixed stars" are not fixed. We already knew that they had their own "proper" motions, but we can now see that the entire celestial sphere is distorted as the Sun moves across the sky. The amount is small, only a few milliarcseconds for most directions; but it is measurable by these VLBI techniques, and radio astronomers now need to take into account the warpage of the sky caused by the curved geometry around the Sun as a routine part of their activities.

In recent years, the bending of light has morphed into the gravitational lens, in which a foreground galaxy or cluster of galaxies bends the light from a more distant source, producing multiple images of the source, distorting distant sources into shapes ranging from ellipses to arcs to rings, and magnifying sources. Gravitational lenses are today being used routinely to map the distribution of mass in the universe, both of ordinary matter and dark matter, and study the possible evolution of dark energy. Gravitational lensing has even been used to discover exoplanets. The gravitational lens is truly Einstein's gift to astronomy.

In these examples, gravity is relatively weak, so the deflections are very tiny; however, in the strong gravity near a black hole, the deflection of light can be very large. A striking example of this effect can be seen in the image of the supermassive black hole "Gargantua" in the 2014 movie *Interstellar*. There, we see not only a disk of radiant hot gas orbiting in front of the black hole but also both the top and bottom sides of the disk *behind* the black hole because light from the two surfaces has been bent by up to 90° in passing by the black hole. (Light rays can even circle the black hole a few times before traveling to the observer.) The images in that movie were based on real calculations, carried out by Caltech astrophysicist Kip Thorne and the special effects team Double Negative, of the paths of light rays in the strongly warped space time of the rapidly rotating black hole. (Thorne was an executive producer on the movie and worked with director Christopher Nolan and the actors on many of the scientific aspects of the plot.) But such imagery may soon enter the realm of science. The Event Horizon Telescope[6] is a project to link together an array of radio telescopes to produce an image of the four-million-solar-mass black hole at the center of the Milky Way with enough angular resolution to see precisely the kinds of strong-gravity bending of light that moviegoers saw in *Interstellar*. Such observations could provide new tests of general relativity in the "strong-gravity" regime.

III. Geometry Warps Time

Soon after turning his attention to the problem of incorporating gravity into special relativity, Einstein had what he later called his "happiest thought." Because bodies appear to fall in a gravitational field with the same acceleration, irrespective of their mass or internal constitution, a person in free fall would sense no gravity. Conversely, a person in an accelerated craft in distant space would imagine that he or she was in a gravitational field. Einstein called this the "principle of equivalence" between gravity and acceleration. He then noted that a light signal emitted from the floor of such an accelerated spacecraft would be received at the ceiling with a frequency shifted to the red (or to lower frequencies), because during the flight of the signal, the ceiling would have accelerated upward to a higher velocity, leading to an apparent Doppler shift of the signal's frequency. According to the principle of equivalence, the same effect should therefore occur in a laboratory at rest on Earth. The effect emerged naturally from the full theory of general relativity, and Einstein came to view the "gravitational redshift" as a crucial test of his theory.

In particular, light emitted by atoms at the surface of the Sun should experience a redshift of its frequency as it propagates toward the Earth, and numerous observers took it upon themselves to test this prediction. However, in 1917, Charles E. St. John of the Mount Wilson Observatory in California reported *no* relativistic shift of spectral lines from the Sun.[7] This result apparently had a negative impact on Einstein's candidacy for the Nobel Prize that year. The prize would not be awarded to him until 1921, and then only for the photoelectric effect, not for any of his relativistic theories.

Together with uncertainties about the bending of light, the failure to measure the redshift had an effect that would last for decades. They were seized upon by some as reason to doubt the theory. For example, the renowned mathematician Alfred North Whitehead produced an alternative theory of gravity in 1922 designed to retain the flat spacetime of special relativity while providing an "action-at-a-distance" tensor potential that would give the correct deflection of light and orbital motion of particles.[8] It predicted no gravitational redshift effect, in accord with the observations of the day.

Unfortunately, the measurement of the solar redshift is not simple. Solar spectral lines are subject to the "limb effect," a variation of spectral line wavelengths between the center of the solar disk and its edge or "limb"; this effect is actually a Doppler shift caused by complex convective and turbulent motions in the solar photosphere and lower chromosphere. Wavelength shifts caused by pressure are also important for certain elements. Truly reliable measurements of the solar redshift would not be made until 1962.

The first true test of the gravitational redshift effect was the classic Pound-Rebka experiment of 1960, which measured the frequency shift of gamma-ray photons from a radioactive isotope of iron (^{57}Fe) as they ascended or descended the Jefferson Physical Laboratory tower at Harvard University.[9] The most precise measurement of the effect to date comes from a space mission known as Gravity Probe A, a joint project of NASA and the Smithsonian Astrophysical Observatory, which compared the rates of two identical hydrogen maser atomic clocks, one on a suborbital rocket launched to a peak altitude of 10,000 km and the other on the ground.[10] The results agreed with the prediction to a few parts in 10,000.

The gravitational frequency shift today has important practical consequences via satellite-based navigational systems, such as GPS. Because of the gravitational redshift effect (with a small offsetting contribution from special relativity), the atomic clocks aboard the 24 satellites of the American GPS system tick faster than atomic clocks on the ground by about 39,000 nanoseconds per day. This difference in rates is huge compared to the 50-nanosecond accuracy required by the system, and thus general relativity *must* be (and is) taken into account for GPS to function properly.[11] This is a very welcome (and quite possibly the only) practical application of general relativity!

Since the 1980s, the effect of gravity on time has been tested in many different ways and to exquisite precision using the latest breakthroughs in precision clock technology, based on ultracold atoms, Bose-Einstein condensates, and atom interferometry. A striking example is an experiment that was done at the University of Colorado in 2010, in which the difference in the rate of time induced by gravity was measured between two clocks based on cold aluminum ions, separated in height by only one third of a meter![12] The European ACES/PHARAOH project hopes to measure the frequency shift between a set of cold-atom clocks on the International Space Station and on the ground, respectively, to parts-per-million precision. Launch is scheduled for August 2017.

IV. Geometry Moves Mass

Newtonian gravity moves mass, of course, so here we are addressing the additional effects that result from general relativity. One of these—the perihelion advance of Mercury—was an immediate success for Einstein and was regarded by him as the third empirical pillar of general relativity.

Yet it did not seem to play as large a role in the early debates over the validity of general relativity as did the deflection of light, in part because the problem of Mercury was an old one, dating back to the mid-1850s. It originated with Urbain Jean Joseph Le Verrier, recently appointed to the directorship of the Observatory of Paris because of his triumphant prediction of the existence of a new planet in the solar system (subsequently named Neptune) based on his studies of anomalies in the orbit of Uranus. Le Verrier announced the "problem of Mercury" around 1859. The perihelion of that planet's orbit—its point of closest approach to the Sun—was known to advance or "precess" at a rate of 575 arcseconds per century. Le Verrier reasoned that this phenomenon was the result of the perturbing effects of the other planets in the solar system, but his calculations could account for only 531 arcseconds, leaving 43 arcseconds unexplained. The natural solution for Le Verrier and his contemporaries was to propose the existence of another planet in an orbit between Mercury and the Sun, whose orbit and mass could be easily chosen to give the desired result. They even assigned the planet the name of Vulcan, after the Roman god of fire, despite having no real evidence of its existence. Although numerous "sightings" of Vulcan were reported during the last decades of the nineteenth century, no credible evidence for the planet was ever produced.

Einstein was well aware of the perihelion problem and in fact used the effect as a way to test the earlier provisional theories that he developed between 1911 and 1915 on the way to the full theory of general relativity. In November 1915, as he was homing in on the final version, he discovered that his latest theory accounted for the missing 43 arcseconds per century. He later wrote to a friend that on obtaining this result, he so was beside himself "in joyous excitement" that he thought he was having a heart attack.

Today, this test of general relativity is very precise. Through a combination of improved values for the orbits and masses of the planets and major asteroids, accurate computer codes for calculating their perturbing effects, and improved measurements of the orbit of Mercury itself, notably from radar tracking of the recent Mercury Messenger orbiter, we now know the value that must be accounted for by one's favorite theory of gravity is 42.98 ± 0.001 arcseconds per century. The prediction of general relativity is 42.98 arcseconds per century.

This and other general relativistic effects on orbits are playing important roles both in testing general relativity and in astrophysics.[13] Solar-system tests of general relativity involving laser ranging to the Moon, radar tracking of interplanetary spacecraft, or laser tracking of Earth-orbiting satellites must account for various relativistic orbital effects. The pericenter advance and the orbital decay induced by gravitational radiation reaction play central roles in the orbits of binary pulsar systems, such as the famous "Hulse-Taylor" binary pulsar, the "double pulsar," and the recently discovered pulsar in a triple system with two companion white dwarf stars. Orbital effects due to general relativity must be calculated to high orders in an approximation to general relativity, known as "post-Newtonian" theory, to determine the orbits of binary systems of neutron stars or black holes with sufficient accuracy to provide a proper basis for data analysis at gravitational-wave detectors (see Section V). Finally, although such relativistic effects as the pericenter advance may be very small, over long periods of time,they may have significant effects, for example on the long-term evolution of clusters of stars around supermassive black holes at the centers of galaxies.

V. Geometry Makes Waves

The idea that there might be waves associated with gravity predates general relativity somewhat, as physicists around the turn of the twentieth century attempted to construct gravitation theories modeled after Maxwell's theory of electrodynamics, which famously predicted electromagnetic waves. The first calculation of such waves within general relativity was done in 1916 by Einstein himself. Unfortunately, that initial paper was full of errors, both conceptual and calculational; a later paper in 1918 corrected most of the errors but still got the formula for the energy flux radiated by a slowly moving source wrong by a factor of two. (The error was later noted and corrected by Eddington). But there were significant conceptual difficulties regarding the true nature of gravitational waves. The equations had multiple solutions, of which only two, apparently, produced real physical effects, whereas the others appeared to be waves of the coordinates used to describe the solutions. Eddington made the unfortunate remark that the latter waves "propagate with the speed of thought,"[14] lending an air of confusion and uncertainty to the whole enterprise. In 1936, Einstein and his assistant Nathan Rosen thought that they had a proof that gravitational waves could *not* exist in general relativity, but an anonymous referee of their paper pointed out that they had misunderstood the nature of the coordinates they were using.[b] Einstein and Rosen then rewrote the paper with the opposite conclusion, displaying an exact solution of the field equations for cylindrical gravitational waves.[15]

These kinds of conceptual problems with gravitational waves were not fully cleared up until the late 1950s, with the work of Felix Pirani, Hermann Bondi, and their collaborators. They elucidated in clear, unambiguous language what the physical content of gravitational waves was, showing that (a) the waves carry energy and angular

b Einstein was so angry that the Physical Review had sent their paper to a referee—a new innovation at the time—that he withdrew the paper and never published in that journal again. The mystery surrounding the anonymous referee was solved in 2005, when Daniel Kennefick convinced the editors of the Physical Review to open their archives, revealing that the referee was the well-known Princeton cosmologist Howard P. Roberston.[15]

momentum, (b) the energy and angular momentum of a source decrease as a result of the emitted radiation, and (c) passing waves cause particles to move in a well-defined and measurable manner.

Meanwhile, a young experimental physicist named Joseph Weber began to think about how to detect gravitational waves and by the late 1960s had built two detectors, one at the University of Maryland and one at Argonne National Laboratory near Chicago. The detectors consisted of cylinders of solid aluminum, each weighing about a ton and suspended by fine wires to attempt to isolate them from surrounding vibrations. Sensing devices were attached to the bars to detect the vibrations that would be induced by a passing gravitational wave. Since any such cylinder has an ever-present noisy background of vibrations induced by local disturbances and thermal fluctuations, a vibration would be considered a candidate for a gravitational wave only if it were stronger than the background noise and appeared essentially simultaneously in both bars.

In 1969 and 1970, Weber announced that he had detected gravitational waves and that they appeared to originate at the center of our galaxy.[16,17] This finding caused a sensation, and had a major impact on research in general relativity. On the theoretical side, it spurred important new research on possible astrophysical sources for gravitational radiation. Unfortunately, the signal strengths that Weber claimed to be detecting were many orders of magnitude larger than any plausible source that could be imagined. It also spurred new work on the empirical foundations of general relativity because if there were no plausible sources, perhaps Einstein was wrong.[c] On the experimental side, it sparked the development of independent detectors in an effort to validate or disprove Weber's results. This effort had the important effect of bringing into the field experimentalists trained in other branches of physics, such as William Fairbank (low temperature physics), Edoardo Amaldi (nuclear physics), Richard Garwin (nuclear physics and magnetic resonance), Ronald Drever (lasers), Vladimir Braginsky (precision measurements), and J. Anthony Tyson (astronomy). Over time, this new experimental program changed the nature of the field, which had been hitherto almost completely dominated by theorists. These new detectors, mostly based on Weber's original cylinders, eventually reached much better sensitivities than Weber's bars yet failed to detect any gravitational waves. By 1980, a consensus emerged that Weber had not detected waves, although there was no "smoking gun" pointing to what he had actually done wrong.

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c My own career traces back to this event because following Weber's announcement, my Ph.D. advisor, Kip Thorne, assigned to me the problem of studying how well general relativity agreed with experiment.

Meanwhile, many of these researchers new to the field began to turn their attention to a different way to detect gravitational waves—using laser interferometry. The idea of this technique is to create a beam of light from a laser, split it into two beams, send each beam to mirrors at the ends of two long arms perpendicular to each other, and then recombine the beams at the original beam splitter. By detecting the tiny shifts in the relative phases of the two beams via the change from constructive to destructive interference, one can measure the changes in distance between the beam splitter and the distant mirrors. Because a relative change in length of the arms of only one half of the wavelength of light converts the output from constructive to destructive interference, laser interferometry can be a very precise tool for measuring tiny distance changes.

To get from these conceptual ideas to the kilometer-scale laser interferometric observatories of today required clever design insights; considerable technical development in lasers, mirrors, and seismic isolation; and significant expenditures of funds provided by the generous taxpayers of several countries. Today, a worldwide network of such observatories exists: the two detectors of the Laser Interferometric Gravitational-wave Observatory (LIGO) in the United States, the Virgo Observatory in Italy, and the GEO-600 Observatory in Germany. The underground KAGRA Observatory in Japan is nearing completion, and plans are moving forward for the LIGO-South observatory in India.

Between 2010 and 2015, the LIGO observatories underwent a major upgrade to improve their sensitivities and then resumed operation in the summer of 2015. (The Virgo observatory underwent a similar upgrade but is about a year behind LIGO in schedule). On September 14, 2015, a signal was detected by LIGO[18] and was designated GW150914. It was received first in the Livingston, Louisiana detector and then, seven milliseconds later, in the Hanford, Washington detector, indicating that the source was in the southern sky. The two signals were significantly stronger than the background noise and were virtually the same, cycle by cycle. They were characteristic of a "chirp," a signal with rising frequency and rising amplitude, the kind of signal expected from the final inspiral of two compact stars, either neutron stars or black holes. A detailed comparison of the two signals with theoretical predictions of general relativity using a combination of analytic formulae and numerical simulations revealed that (a) the two bodies were black holes of 36 and 29 solar masses, respectively; (b) the final black hole had a mass of 62 solar masses; and (c) the source was 1.3 billion light years away. During the 0.2 seconds when the waves were the strongest, the system was converting those three solar masses into pure energy, representing a power that exceeds that of all the stars in the observable universe.

In addition to being the first direct detection of gravitational waves, this is the first detection of a binary system of black holes. Strong evidence already existed for binary systems containing neutron stars from the many "binary pulsar" systems that have been discovered via radio astronomy, but there was no direct evidence for binary black holes (mainly because, for the most part, they emit only gravitational waves). In fact, the masses inferred for GW150914 were rather large compared to the five to 15 solar masses characteristic of those black holes that have been detected orbiting normal stars using X-ray astronomy. The LIGO discovery has already presented a challenge to astrophysicists to account for such large masses. The detailed shape of the waveform was found to agree with the prediction of general relativity and also was in accord with the theory's prediction that the waves propagate at the same speed independently of wavelength, indicating that the waves are associated with a "massless" field, just as in the case of electrodynamics.[19]

On December 26, 2015, a second binary black hole system was detected, with 7.5 and 14.2 solar masses. This finding suggests that there may well be many more detections to come and that "gravitational-wave astronomy" will become an important new way to observe the universe.

VI. Conclusions

At the centenary of general relativity, we see that Einstein's great theory has been tested in many ways and to high precision—and it has passed every test. Today, it is an active and vigorous branch of physics and astronomy. As we look toward the second century of general relativity, the central themes are going to be (a) testing general relativity in the strong-gravity regime near black holes and neutron stars, going beyond the weak-gravity conditions of the solar system to provide quantitative tests in a new regime; (b) using gravitational waves as a tool to test general relativity, not only by studying the properties of the waves (i.e., their speed and polarization modes) but also by using them to study their sources under strong-gravity conditions; and (c) testing general relativity on the largest scales, those of the observable universe itself.

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